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Algebra III: Blizzard Bag #1

Exponential and Logarithm Functions

- Students need to complete the following assignment, which will aid in review for the end of course exam.

- Look back on previous notes from the sections covered, if you need assistance completing the work.

- All of the ODD problems need to be completed.

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3-4: *Exponential and Logarithmic Equations*

Solve Exponential Equations **One-to-One Property of Exponential Functions:** For $b > 0$ and $b \neq 1$, $bx = by$ if and only if $x = y$. This property will help you solve exponential equations. For example, you can express both sides of the equation as an exponent with the same base. Then use the property to set the exponents equal to each other and solve.
If the bases are not the same, you can *exponentiate* each side of an equation and use logarithms to solve the equation.

Example 1

a. Solve $4^{x-1} = 16^x$.

$4^{x-1} = 16^x$	Original equation
$4^{x-1} = (4^2)^x$	$16 = 4^2$
$4^{x-1} = 4^{2x}$	Power of a Power
$x - 1 = 2x$	One-to-One Property
$-1 = x$	Subtract x from each side.

b. Solve $e^{2x} - 3e^x + 2 = 0$.

$e^{2x} - 3e^x + 2 = 0$	Original equation
$u^2 - 3u + 2 = 0$	Write in quadratic form.
$(u - 2)(u - 1) = 0$	Factor.
$u = 2$ or $u = 1$	Solve.
$e^x = 2$ or $e^x = 1$	Substitute for u.
$x = \ln 2$ or 0	Take the natural logarithm of each side

Example 2: Solve each equation. Round to the nearest hundredth if necessary.

a. $3^x = 19$

$\log 3^x = \log 19$	Take the log of both sides.
$x \log 3 = \log 19$	Power Property
$x = \frac{\log 19}{\log 3}$	Divide each side by log 3.
$x \approx 2.68$	Use a calculator. Check this solution in the original equation.

b. $e^{8x+1} - 6 = 1$

$e^{8x+1} = 7$	Add 6 to both sides.
$\ln e^{8x+1} = \ln 7$	Take the ln of both sides.
$(8x + 1) \ln e = \ln 7$	Power Property
$8x + 1 = \ln 7$	$\ln e = 1$
$8x = \ln 7 - 1$	Subtract 1 from each side.
$x = \frac{\ln 7 - 1}{8} \approx 0.12$	Divide by 8 and use a calculator.

Exercises

Solve each equation. Round to the nearest hundredth.

1. $9^x = 3^{3x-4}$

2. $\left(\frac{1}{4}\right)^{2x-1} = \left(\frac{1}{8}\right)^{11-x}$

3. $4^{3x-2} = \frac{1}{2}^{2x}$

4. $2e^{2x} + 12e^x - 54 = 0$

5. $9^{2x} = 12$

6. $2.4e^{x-6} = 9.3$

7. $3^{2x} = 6^{x-1}$

8. $e^{19x} = 23$

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Solve Logarithmic Equations **One-to-One Property of Exponential Functions**

For $b > 0$ and $b \neq 1$, $\log_b x = \log_b y$ if and only if $x = y$.

This property will help you solve logarithmic equations. For example, you can express both sides of the equation as a logarithm with the same base. Then convert both sides to exponential form, set the exponents equal to each other and solve.

Example 1: Solve $2\log_5 4x - 1 = 11$.

$2\log_5 4x - 1 = 11$	Original equation
$2\log_5 4x = 12$	Add 1 to each side.
$\log_5 4x = 6$	Divide each side by 2.
$4x = 5^6$	Write in exponential form. (Use 5 as the base when exponentiating.)
$x = \frac{5^6}{4}$	Divide each side by 4.
$x = 3906.25$	Use a calculator.

Example 2: Solve $\log_2(x - 6) = 5 - \log_2 2x$.

$\log_2 (x - 6) = 5 - \log_2 2x$	Original equation
$\log_2 (x - 6) + \log_2 2x = 5$	Rearrange the logs.
$\log_2 (2x(x - 6)) = 5$	Product Property
$2x(x - 6) = 2^5$	Rewrite in exponential form.
$2x^2 - 12x - 32 = 0$	Expand.
$2(x - 8)(x + 2) = 0$	Factor.
$x = 8$ or -2	Solve.

CHECK

$x = -2$ $\log_2 (-2 - 6) = 5 - \log_2 [2(-2)]$

yields logs of negative numbers.

Therefore, –2 is extraneous.

$x = 8$ $\log_2 (8 - 6) = 5 - \log_2 [2(8)]$

$\log_2 2 = 5 - \log_2 16$, which is true.

Therefore, $x = 8$.

Exercises

Solve each logarithmic equation.

1. $\log 3x = \log 12$

2. $\log_{12} 2 + \log_{12} x = \log_{12} (x + 7)$

3. $\log (x + 1) + \log (x - 3) = \log (6x^2 - 6)$

4. $\log_3 3x = \log_3 36$

5. $\log (16x + 2) + \log (20x - 2) = \log (319x^2 + 9x - 2)$

6. $\ln x + \ln (x + 16) = \ln 8 + \ln (x + 6)$

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3-3: *Properties of Logarithms*

Properties of Logarithms Since logarithms and exponents have an inverse relationship, they have certain properties that can be used to make them easier to simplify and solve.

If b, x, and y are positive real numbers, $b \neq 1$, and p is a real number, then the following statements are true.

- $\log_b xy = \log_b x + \log_b y$ Product Property
- $\log_b \frac{x}{y} = \log_b x - \log_b y$ Quotient Property
- $\log_b x^p = p \log_b x$ Power Property

Example 1: Evaluate $3 \log_2 8 + 5 \log_2 \frac{1}{2}$.

$3 \log_2 8 + 5 \log_2 \frac{1}{2} = 3 \log_2 2^3 + 5 \log_2 2^{-1}$	$8 = 2^3$; $2^{-1} = \frac{1}{2}$
$= 3(3 \log_2 2) + 5(-\log_2 2)$	Power Property
$= 3(3)(1) + 5(-1)(1)$	$\log_x x = 1$
$= 4$	Simplify.

Example 2: Expand $\ln \frac{8x^5}{3y^2}$.

$\ln \frac{8x^5}{3y^2} = \ln 8x^5 - \ln 3y^2$	Quotient Property
$= \ln 8 + \ln x^5 - \ln 3 - \ln y^2$	Product Property
$= \ln 8 + 5 \ln x - \ln 3 - 2 \ln y$	Power Property

Exercises

1. Evaluate $2 \log_3 27 + 4 \log_3 \frac{1}{3}$.

Expand each expression.

2. $\log_3 \frac{5r^5}{\sqrt[3]{t^2}}$

3. $\log \frac{(a-2)(b+4)^6}{9(b-2)^5}$

Condense each expression.

4. $11 \log_9 (x-3) - 5 \log_9 2x$

5. $\frac{3}{4} \ln (2h - k) + \frac{3}{5} \ln (2h + k)$

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Change of Base Formula If the logarithm is in a base that needs to be changed to a different base, the **Change of Base Formula** is required.

For any positive real numbers *a*, *b*, and *x*, $a \neq 1$, $b \neq 1$, $\log_b x = \frac{\log_a x}{\log_a b}$.

Many non-graphing calculators cannot be used for logarithms that are not base *e* or base 10. Therefore, you will often use this formula, especially for scientific applications. Either of the following forms will provide the correct answer.

$$\log_b x = \frac{\log x}{\log b} \qquad \log_b x = \frac{\ln x}{\ln b}$$

Example: Evaluate each logarithm.

a. $\log_2 7$

$\log_2 7 = \frac{\ln 7}{\ln 2}$ Change of Base Formula

≈ 2.81 Use a calculator.

b. $\log_{\frac{1}{3}} 10$

$\log_{\frac{1}{3}} 10 = \frac{\log 10}{\log \frac{1}{3}}$ Change of Base Formula

≈ -2.10 Use a calculator.

Exercises

Evaluate each logarithm.

1. $\log_{32} 631$

2. $\log_3 17$

3. $\log_7 1094$

4. $\log_6 94$

5. $\log_5 256$

6. $\log_9 712$

7. $\log_6 832$

8. $\log_{11} 47$

9. $\log_3 9$

10. $\log_8 256$

11. $\log_{12} 4302$

12. $\log_{0.5} 420$

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3-2: *Logarithmic Functions*

Logarithmic Functions and Expressions The inverse relationship between logarithmic functions and exponential functions can be used to evaluate logarithmic expressions.

If $b > 0$, $b \neq 1$, and $x > 0$, then

Logarithmic Form		**Exponential Form**
$log_b\ x = y$	if and only if	$b^y = x$
base ↑ exponent ↑		base ↑ exponent ↑

The following properties are also useful.

$\log_b 1 = 0$ $\log_b b = 1$ $\log_b b^x = x$ $b^{\log_b x} = x, x > 0$

Example 1: Evaluate each logarithm.

a. $\mathbf{\log_5 \frac{1}{25}}$

$\log_5 \frac{1}{25} = y$	Let $\log_5 \frac{1}{25} = y$.
$5^y = \frac{1}{25}$	Write in exponential form.
$5^y = 5^{-2}$	$\frac{1}{25} = 5^{-2}$
$y = -2$	Equality Prop. of Exponents

Therefore, $\log_5 \frac{1}{25} = -2$ because $5^{-2} = \frac{1}{25}$.

b. $\mathbf{\log_3 \sqrt{3}}$

$\log_3 \sqrt{3} = y$	Let $\log_3 \sqrt{3} = y$.
$3^y = \sqrt{3}$	Write in exponential form.
$3^y = 3^{\frac{1}{2}}$	$3^{\frac{1}{2}} = \sqrt{3}$
$y = \frac{1}{2}$	Equality Prop. of Exponents

Therefore, $\log_3 \sqrt{3} = \frac{1}{2}$ because $3^{\frac{1}{2}} = \sqrt{3}$.

Example 2: Evaluate each expression.

a. $\mathbf{\ln e^7}$

$\ln e^7 = 7$ $\ln e^x = x$

b. $\mathbf{e^{\ln 5}}$

$e^{\ln 5} = 5$ $e^{\ln 5} = x$

c. $\mathbf{10^{\log 13}}$

$10^{\log 13} = 13$ $10^{\log x} = x$

Exercises

Evaluate each logarithm.

1. $\log_7 7$

2. $10^{\log 5x}$

3. $3^{\log_3 2}$

4. $\log_6 36$

5. $\log_3 \frac{1}{81}$

6. $e^{\ln x^2}$

7. FINANCIAL LITERACY Ms. Dasilva has $3000 to invest. She would like to invest in an account that compounds continuously at 6%. Use the formula $\ln A - \ln P = rt$, where A is the current balance, P is the original principal, r is the rate as a decimal, and t is the time in years. How long will it take for her balance to be $6000?